An adaptive time domain approach to characterize dispersive elastodynamic media

Reza Abedi

Mechanical, Aerospace & Biomedical Engineering, University of Tennessee Knoxville (UTK) / Space Institute (UTSI), 411 B. H. Goethert Parkway, Tullahoma, TN 37388

The response of many materials in wave propagation problems, e.g. electromagnetics, acoustics, and elastodynamics, is dispersive. That is, material constitutive parameters and the speed of wave propagation depend on frequency. Viscoelastic materials are a classical example from elastodynamics. Furthermore, in composites and other heterogeneous materials the interaction of waves with heterogeneities, when the wavelength is not drastically larger than the length scale of heterogeneities, results in a complex and dispersive response. In fact, this is the basis of the design of metamaterials which can exhibit exotic dynamics response, such as effective negative stiffness and/or mass density in elastodynamics. Time domain (TD) and frequency domain (FD) methods are two approaches to simulate and characterize dispersive media. FD methods solve an underlying problem for a discrete set of frequencies to characterize the response. For TD approaches the solution to a broad-band, *i.e.* a temporally short, pulse and a subsequent Fourier transform of the solution are sufficient to characterize the response for the range of the frequencies considered. As the problem size increases, that is the number of elements in the spatial domain increases, TD approaches become more efficient due to their linear solution complexity versus number of elements and the need for only one solution.

A mesh adaptive TD approach is presented to obtain reflection and transmission coefficients of a material for a wide range of frequencies. The advanced method of asynchronous spacetime discontinuous Galerkin method is used to obtain the TD response of a unit cell to an incident wave. Adaptive operations in space and time permit very efficient and accurate tracking of wave fronts. By Fourier analysis and inversion of the obtained transmission and reflection coefficients in the frequency domain, we obtain equivalent impedance and wave speed for the slab of material considered. By maintaining the continuity of real part of wavenumber k, as angular frequency ω increases, we ensure that k and wave speed are characterized uniquely. Finally, the corresponding elastic modulus and mass density are obtained from the wave speed and impedance. Multi-layer 1D unit cells and a 2D metamaterial unit cell are analysed and characterized with the proposed TD approach. The effect of adaptive tolerance and various parameters of TD signal on the accuracy of derived dispersive elastic moduli and mass density is investigated.